Extra Practice Problems 1

Over the course of the week, we will be releasing a series of practice problems to help you prepare for the first midterm exam. Here's an initial set of extra practice problems you can use to practice your proofwriting and first-order logic skills. We'll release solutions on Wednesday. In the meantime, feel free to ask questions on Piazza or stop by office hours if you have questions.

Problem One: First-Order Logic

Here's some more practice with translating statements into first-order logic.

   i.   Given the predicates

        Person(p), which states that p is a person, and
        ParentOf(p_1, p_2), which states that p_1 is the parent of p_2,

        write a statement in first-order logic that says "someone is their own grandparent." (Paraphrased from an old novelty song.)

   ii. Given the predicates

        Set(S), which states that S is a set, and
        $x \in y$, which states that x is an element of y,

        write a statement in first-order logic that says "for any sets S and T, the set $S \Delta T$ exists."

   iii. Given the predicates

        Set(S), which states that S is a set;
        $x \in y$, which states that x is an element of y;
        Natural(n), which states that n is a natural number; and
        $x < y$, which states that x is less than y,

        write a statement in first-order logic that says "if S is a nonempty subset of $\mathbb{N}$, then S contains a natural number that's smaller than all the other natural numbers in S." (This is called the well-ordering principle.)

Problem Two: First-Order Negations

For each of the statements you came up with in part (i) of this problem, negate that statement and push the negation as deep as possible, along the lines of what you did in Problem Set Two. Then, for each statement, translate it back into English and make sure you see why it's the negation of the original formula.

Problem Three: Set Theory

Below are a number of claims about sets. For each claim, decide whether the statement is true or false. If it's true, prove it. If it's false, disprove it.

    i.   For any sets A, B, and C, if $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$.

    ii. For any sets A, B, and C, if $A \subsetneq B$ and $A \subsetneq C$, then $A \subsetneq B \cap C$. (Here, the notation $A \subsetneq B$ means "$A \subseteq B$ and $A \neq B$.")

    iii. For any set A, if $A \subsetneq \emptyset$, then $137 \in A$.

Problem Four: Latin Squares

A **Latin square** is an $n \times n$ grid such that every natural number between 1 and n, inclusive, appears exactly once on each row and column. A **symmetric Latin square** is a Latin square that is symmetric across the main diagonal from the upper-left corner to the lower-right corner. Specifically, the elements at positions (i, j) and (j, i) are always the same.

Prove that in any $n \times n$ symmetric Latin square, where n is even, there is at least one number between 1 and n that appears nowhere on the diagonal.